

# A TWO-MODE DIELECTRIC ROD RESONATOR METHOD FOR MEASURING SURFACE IMPEDANCE OF HIGH-T<sub>C</sub> SUPERCONDUCTORS

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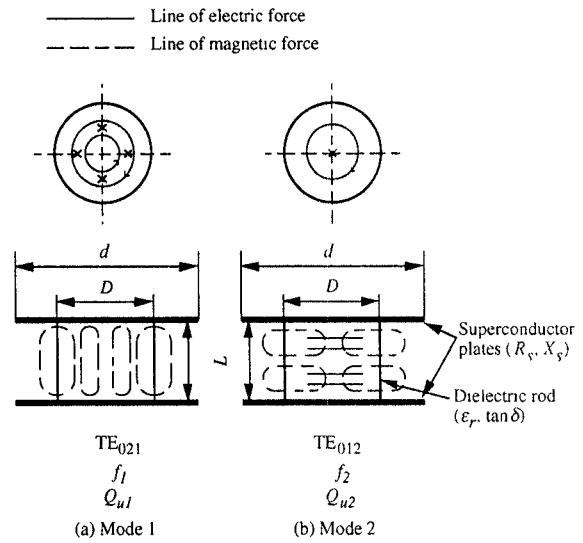
## ABSTRACT

A novel technique is proposed to measure the surface impedance  $Z_s$  of high-T<sub>c</sub> superconductors at microwave frequency by using two resonant modes in a dielectric rod resonator, the TE<sub>021</sub> and TE<sub>012</sub> modes. The repeatability of measurement is superior to the conventional method using two dielectric resonators.

## INTRODUCTION

Measurement of the surface impedance  $Z_s = R_s + jX_s$ , where  $R_s$  is the surface resistance and  $X_s$  is the surface reactance, of high-T<sub>c</sub> superconductors at microwave frequency, is essential for the material evaluation and development and for research of superconductor physics. For the  $R_s$  measurements, many papers have been presented. For the  $X_s$  measurements, however, there are few papers [1]-[3], and the measurement accuracies are not sufficient.

In this paper another method is proposed to measure the temperature dependences of  $X_s$  as well as  $R_s$ , simultaneously. In this method, which will be called one-resonator method, two resonant modes in a dielectric rod resonator, the TE<sub>021</sub> and TE<sub>012</sub> modes are used [4], [5]. It was verified an accurate measurement of  $R_s$  can be realized because the measurements for these two modes can be performed simultaneously at a given temperature [5]. A high accuracy can be expected also for the  $X_s$  measurements.



$$\varepsilon_{rp} = \left( \frac{c}{\pi D f_p} \right)^2 (u_p^2 + v_p^2) + 1$$

where  $v_p^2 = \left( \frac{\pi D f_p}{c} \right)^2 \left\{ \left( \frac{c p}{2 L f_p} \right)^2 - 1 \right\}$  for  $p=1$  or 2

$$D = D_0 [1 + \tau_\alpha (T - T_0)] \quad L = L_0 [1 + \tau_\alpha (T - T_0)]$$

$$\tan \delta_0 = \frac{B_1 f_1^n f_0 \frac{A_2}{Q_{u2}} - B_2 f_2^n f_0 \frac{A_1}{Q_{u1}}}{B_1 f_1^n f_2 - B_2 f_2^n f_1} \quad \text{at } f_0$$

$$R_s = \frac{f_0^n f_2 \frac{A_1}{Q_{u1}} - f_0^n f_1 \frac{A_2}{Q_{u2}}}{B_1 f_1^n f_2 - B_2 f_2^n f_1} \quad \text{at } f_0$$

where  $n=2$  for superconductor and  $n=0.5$  for normal conductor.

Fig. 1. Field plots of the TE<sub>021</sub> and TE<sub>012</sub> modes in a dielectric rod resonator placed between two parallel conducting plates and formulas to measure the temperature dependences of  $\varepsilon_r$ ,  $\tan \delta$  and  $R_s$ .

## MEASUREMENT PRINCIPLE OF $X_s$

In this method, two resonant modes  $TE_{021}$  and  $TE_{012}$  are used in a dielectric rod resonator having  $\epsilon_r$ , diameter  $D$ , and length  $L$ , which is placed between two parallel conductor plates having  $Z_s$  and diameter  $d$ . Field plots of these modes are shown in Fig. 1, together with measurement formulas [5]. The  $\epsilon_{rp}$  is obtained from the measured  $f_p$  value and the values of  $\tan\delta_0$  and  $R_{s0}$  at  $f=f_0$  are obtained from the measured unloaded  $Q$  values  $Q_{u1}$  and  $Q_{u2}$ .

Using the formula for  $\epsilon_{rp}$ , which is derived under the condition  $R_s=X_s=0$ , we can calculate  $\epsilon_{r1}$  from the measured resonant frequency  $f_1$  for the  $TE_{021}$  mode ( $p=1$ ) and similarly  $\epsilon_{r2}$  from  $f_2$  for the  $TE_{012}$  mode ( $p=2$ ). The measured values  $\epsilon_{r1}$  and  $\epsilon_{r2}$  values are different each other due to the different effect of  $X_s$ , depending strongly on the modes. An actual relative permittivity  $\epsilon_{act}$  can be calculated by taking the effect of  $X_s$  to  $\epsilon_r$  into account; that is,

$$\epsilon_{act} = \epsilon_{r1} + \Delta\epsilon_{r1} = \epsilon_{r2} + \Delta\epsilon_{r2} \quad (1)$$

where  $\Delta\epsilon_{r1}$  and  $\Delta\epsilon_{r2}$  are correction terms due to  $X_{s1}$  for the  $TE_{021}$  mode and due to  $X_{s2}$  for the  $TE_{012}$  mode, respectively.

Derivation of  $\Delta\epsilon_r$  as a function of  $X_s$  is described below. Fig. 2 shows an analytical model used in this analysis on the assumption of  $d=\infty$  and  $\tan\delta=0$ . From an equivalent circuit of this resonator shown in Fig. 2. (b), the resonance condition is given by

$$Z_s + j Z_\beta \tan \beta L = - \frac{Z_s}{Z_\beta} (Z_s + j Z_\beta \tan \beta L) \quad (2)$$

where

$$Z_\beta = \frac{\omega\mu}{\beta} \quad Z_s = R_s + jX_s = \frac{j\omega\mu}{\gamma} \quad (3)$$

and  $\mu=\mu_0=4\pi \times 10^{-7}$  H/m,  $Z_\beta$  and  $\beta$  are the characteristic impedance and phase constant in a dielectric waveguide, and  $\gamma$  is the propagation constant in superconductor. From Eqs. (2) and (3),

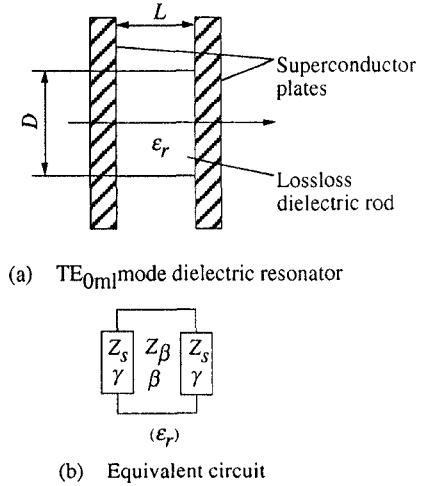


Fig. 2. Analytical model.

$\Delta\epsilon_r$  is derived by the perturbation technique; that is,

$$\Delta\epsilon_{rp} = -2 \left(1 + W_p\right) \left(\frac{\beta_p}{k_p}\right)^2 \frac{2X_{sp}}{\omega_p \mu L} \quad (4)$$

where

$$W_p = \frac{J_1^2(u_p) \left\{ K_0(v_p)K_2(v_p) - K_1^2(v_p) \right\}}{K_1^2(v_p) \left\{ J_1^2(u_p) - J_0(u_p)K_2(u_p) \right\}} \quad k_p = \frac{\omega_p}{c} \quad (5)$$

When  $X_s$  is assumed to be proportional to  $f$  for superconductor, the surface reactance  $X_{s0}$  at  $f_0$ , which is given arbitrarily near  $f_1$  and  $f_2$ , is obtained from the measured values  $\epsilon_{r1}$  and  $\epsilon_{r2}$  by

$$X_{s0} = \frac{\epsilon_{r2} - \epsilon_{r1}}{(C_1 - C_2)} \quad (6)$$

where

$$C_p = -2 \left(1 + W_p\right) \left(\frac{\beta_p}{k_p}\right)^2 \frac{1}{\pi f_0 \mu L} \quad (7)$$

which is derived from substitution of Eq. (4) into Eq. (1).

Furthermore, based on the two-fluid model [1], the complex conductivity  $\sigma$  can be obtained from measured  $Z_s$  value by

$$\sigma = \sigma_1 - j\sigma_2 = \frac{j\omega\mu_0}{Z_s^2} \quad (8)$$

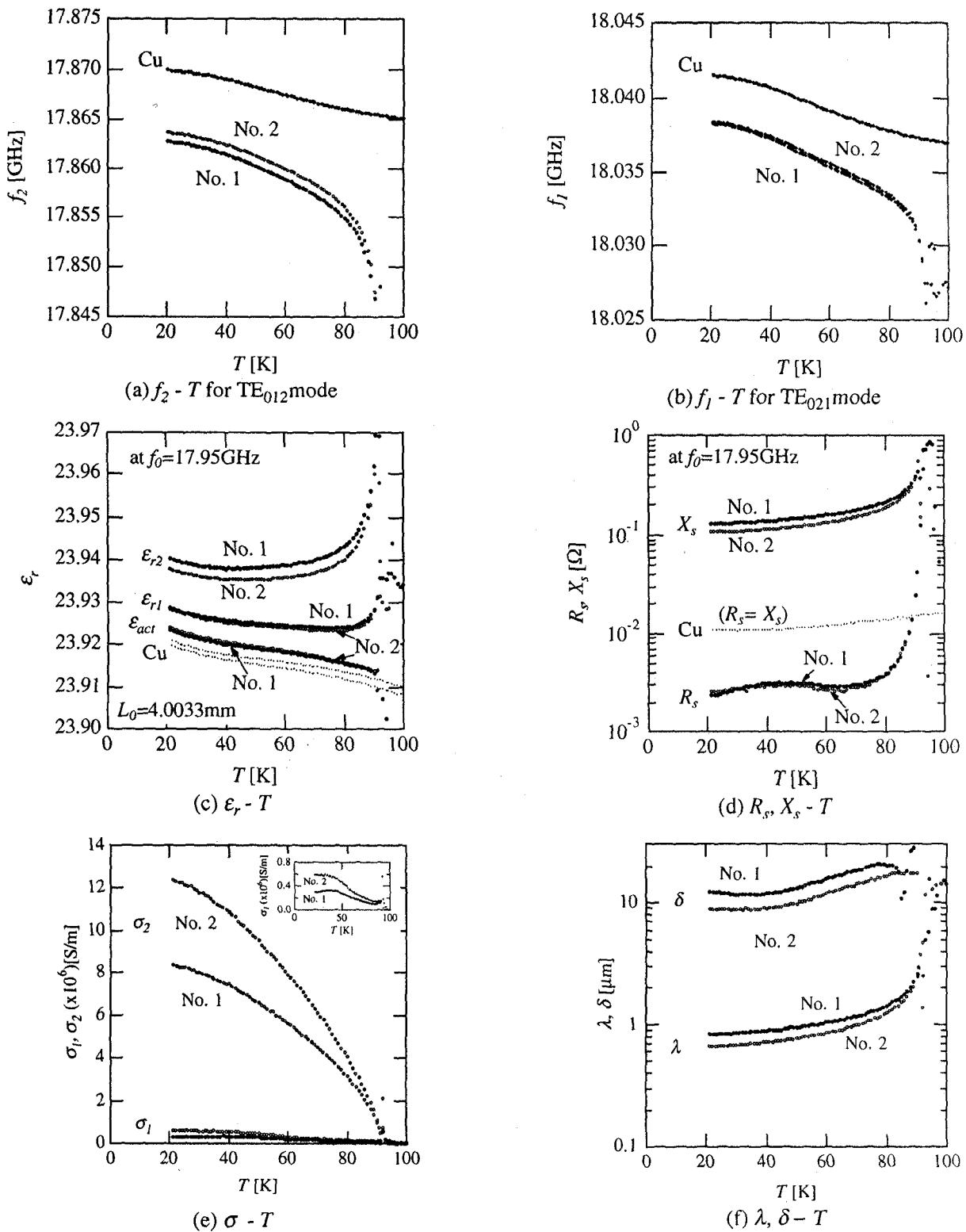


Fig. 3. Measured results for  $\epsilon_r$  and  $\tan\delta$  of a BMT ceramic rod and  $Z_s$  of melt-textured YBCO bulk plates

by one-resonator method.

(Nos. 1 and 2 are for the first and second measurements, respectively.)

and the skin depth  $\delta$  and the penetration length  $\lambda$  are given by

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma_1}} \quad (9)$$

$$\lambda = \sqrt{\frac{1}{\omega\mu_0\sigma_2}} \quad (10)$$

## MEASURED RESULTS

A program was developed to measure the  $T$  dependencies of  $\epsilon_r$ ,  $\tan\delta$  and  $Z_s$  automatically. In comparison with superconductor, at first, a BMT ceramic rod ( $\epsilon_r=24$ ,  $\tau_\alpha=6.3\text{ppm/K}$ , MURATA MFG. CO.) having  $D=6.994\pm0.001\text{mm}$  and  $L=4.003\pm0.001\text{mm}$  [5] was placed between two copper plates of  $d=35\text{mm}$ , which is greater than estimated value 28mm required for measurement. The values of  $f_1$ ,  $Q_{u1}$ ,  $f_2$  and  $Q_{u2}$  were measured as a function of  $T$ . The values of  $\epsilon_r$  were calculated from these measured values, where  $L_0=4.0035\text{mm}$  was used in calculation, so that two  $\epsilon_r$  values take the same values. Furthermore, the actual length of the rod was decided to be  $L_0=4.0033\text{mm}$  by taking the skin depths of two copper plates  $\delta=0.2\mu\text{m}$  into account. The detail is given in [5].

Then the two copper plates were changed into two melt-textured YBCO bulk plates (IMURA MATERIAL R&D CO.) of  $d=33\text{mm}$  [5]. Similarly the values of  $f_1$ ,  $Q_{u1}$ ,  $f_2$  and  $Q_{u2}$  were measured as a function of  $T$ . A similar measurement was repeated.

The  $f_1$  and  $f_2$  for the Cu and YBCO plates are shown in Fig. 3 (a), (b) where the Nos. 1 and 2 indicate in the curves are for the first and second measurements, respectively. The  $\epsilon_r$  values calculated from these  $f_1$  and  $f_2$  values are shown in Fig. 3 (c). The  $\epsilon_{act}$  and  $X_s$  values calculated from the  $\epsilon_{r1}$  and  $\epsilon_{r2}$  values are given in Fig. 3 (c) and (d), respectively. The  $R_s$  values in Fig. 3 (d) have already presented in [5]. The  $\sigma_1$  and  $\sigma_2$  values calculated using the measured  $Z_s$  values are shown in Fig. 3 (e).

Furthermore,  $\lambda$  and  $\delta$  values calculated using  $\sigma_1$  and  $\sigma_2$  values are shown in Fig. 3 (f). The reproducibility of these measured results is considerably good.

## CONCLUSION

It was verified that the one-resonator method is useful to measure the  $T$  dependence of  $Z_s$ , because they can be obtained from only one measurement for these two modes. The excellent reproducibility of measurements can be obtained, compared with the conventional two-resonator method.

## References

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