

A TWO-MODE DIELECTRIC ROD RESONATOR METHOD FOR MEASURING SURFACE IMPEDANCE OF HIGH- T_c SUPERCONDUCTORS

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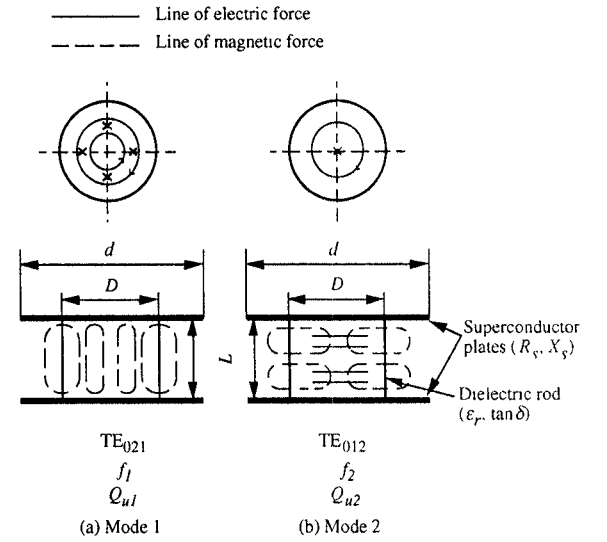
ABSTRACT

A novel technique is proposed to measure the surface impedance Z_s of high- T_c superconductors at microwave frequency by using two resonant modes in a dielectric rod resonator, the TE_{021} and TE_{012} modes. The repeatability of measurement is superior to the conventional method using two dielectric resonators.

INTRODUCTION

Measurement of the surface impedance $Z_s = R_s + jX_s$, where R_s is the surface resistance and X_s is the surface reactance, of high- T_c superconductors at microwave frequency, is essential for the material evaluation and development and for research of superconductor physics. For the R_s measurements, many papers have been presented. For the X_s measurements, however, there are few papers [1]-[3], and the measurement accuracies are not sufficient.

In this paper another method is proposed to measure the temperature dependences of X_s as well as R_s , simultaneously. In this method, which will be called one-resonator method, two resonant modes in a dielectric rod resonator, the TE_{021} and TE_{012} modes are used [4], [5]. It was verified an accurate measurement of R_s can be realized because the measurements for these two modes can be performed simultaneously at a given temperature [5]. A high accuracy can be expected also for the X_s measurements.



$$\epsilon_{rp} = \left(\frac{c}{\pi D f_p} \right)^2 (u_p^2 + v_p^2) + 1$$

$$\text{where } v_p^2 = \left(\frac{\pi D f_p}{c} \right)^2 \left\{ \left(\frac{cp}{2L f_p} \right)^2 - 1 \right\} \quad \text{for } p=1 \text{ or } 2$$

$$D = D_0 [1 + \tau_\alpha (T - T_0)] \quad L = L_0 [1 + \tau_\alpha (T - T_0)]$$

$$\tan \delta_0 = \frac{B_1 f_1^n f_0 \frac{A_2}{Q_{u2}} - B_2 f_2^n f_0 \frac{A_1}{Q_{u1}}}{B_1 f_1^n f_2 - B_2 f_2^n f_1} \quad \text{at } f_0$$

$$R_{s0} = \frac{f_0^n f_2 \frac{A_1}{Q_{u1}} - f_0^n f_1 \frac{A_2}{Q_{u2}}}{B_1 f_1^n f_2 - B_2 f_2^n f_1} \quad \text{at } f_0$$

where $n=2$ for superconductor and $n=0.5$ for normal conductor.

Fig. 1. Field plots of the TE_{021} and TE_{012} modes in a dielectric rod resonator placed between two parallel conducting plates and formulas to measure the temperature dependences of ϵ_r , $\tan \delta$ and R_s .

MEASUREMENT PRINCIPLE OF X_s

In this method, two resonant modes TE_{021} and TE_{012} are used in a dielectric rod resonator having ϵ_r , diameter D , and length L , which is placed between two parallel conductor plates having Z_s and diameter d . Field plots of these modes are shown in Fig. 1, together with measurement formulas [5]. The ϵ_{rp} is obtained from the measured f_p value and the values of $\tan\delta_0$ and R_{s0} at $f=f_0$ are obtained from the measured unloaded Q values Q_{u1} and Q_{u2} .

Using the formula for ϵ_{rp} , which is derived under the condition $R_s=X_s=0$, we can calculate ϵ_{r1} from the measured resonant frequency f_1 for the TE_{021} mode ($p=1$) and similarly ϵ_{r2} from f_2 for the TE_{012} mode ($p=2$). The measured values ϵ_{r1} and ϵ_{r2} values are different each other due to the different effect of X_s , depending strongly on the modes. An actual relative permittivity ϵ_{act} can be calculated by taking the effect of X_s to ϵ_r into account; that is,

$$\epsilon_{act} = \epsilon_{r1} + \Delta\epsilon_{r1} = \epsilon_{r2} + \Delta\epsilon_{r2} \quad (1)$$

where $\Delta\epsilon_{r1}$ and $\Delta\epsilon_{r2}$ are correction terms due to X_{s1} for the TE_{021} mode and due to X_{s2} for the TE_{012} mode, respectively.

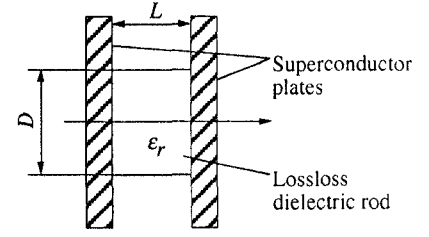
Derivation of $\Delta\epsilon_r$ as a function of X_s is described below. Fig. 2 shows an analytical model used in this analysis on the assumption of $d=\infty$ and $\tan\delta=0$. From an equivalent circuit of this resonator shown in Fig. 2. (b), the resonance condition is given by

$$Z_s + j Z_\beta \tan \beta L = - \frac{Z_s}{Z_\beta} (Z_s + j Z_\beta \tan \beta L) \quad (2)$$

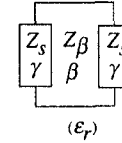
where

$$Z_\beta = \frac{\omega\mu}{\beta} \quad Z_s = R_s + jX_s = \frac{j\omega\mu}{\gamma} \quad (3)$$

and $\mu=\mu_0=4\pi \times 10^{-7}$ H/m, Z_β and β are the characteristic impedance and phase constant in a dielectric waveguide, and γ is the propagation constant in superconductor. From Eqs. (2) and (3),



(a) TE_{0m1} mode dielectric resonator



(b) Equivalent circuit

Fig. 2. Analytical model.

$\Delta\epsilon_r$ is derived by the perturbation technique; that is,

$$\Delta\epsilon_{rp} = -2 \left(1 + W_p \right) \left(\frac{\beta_p}{k_p} \right)^2 \frac{2 X_{sp}}{\omega_p \mu L} \quad (4)$$

where

$$W_p = \frac{J_1^2(u_p) \left\{ K_0(v_p) K_2(v_p) - K_1^2(v_p) \right\}}{K_1^2(v_p) \left\{ J_1^2(u_p) - J_0(u_p) K_2(u_p) \right\}} \quad k_p = \frac{\omega_p}{c} \quad (5)$$

When X_s is assumed to be proportional to f for superconductor, the surface reactance X_{s0} at f_0 , which is given arbitrarily near f_1 and f_2 , is obtained from the measured values ϵ_{r1} and ϵ_{r2} by

$$X_{s0} = \frac{\epsilon_{r2} - \epsilon_{r1}}{(C_1 - C_2)} \quad (6)$$

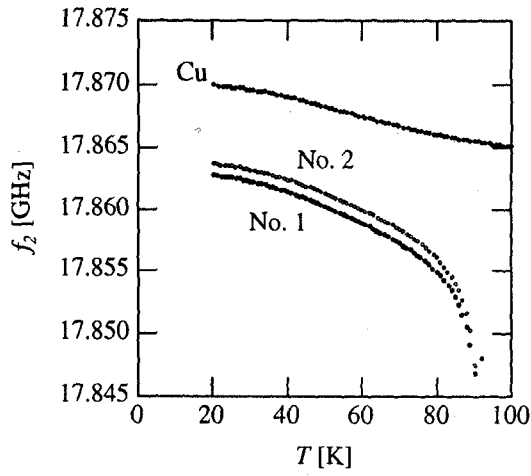
where

$$C_p = -2 \left(1 + W_p \right) \left(\frac{\beta_p}{k_p} \right)^2 \frac{1}{\pi f_0 \mu L} \quad (7)$$

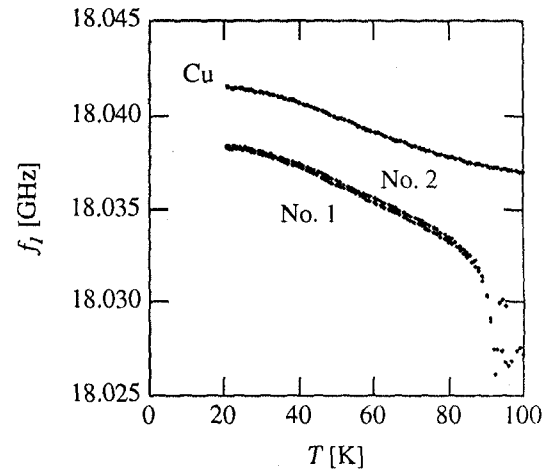
which is derived from substitution of Eq. (4) into Eq. (1).

Furthermore, based on the two-fluid model [1], the complex conductivity σ can be obtained from measured Z_s value by

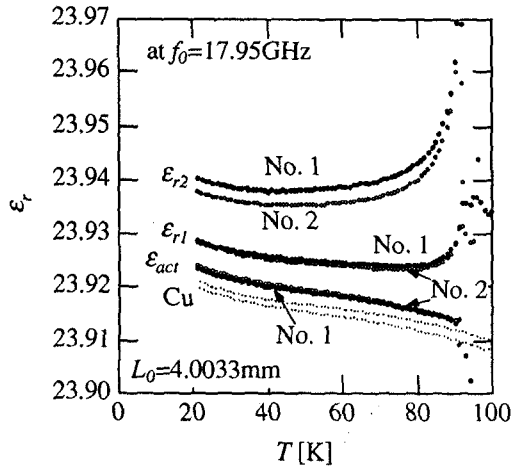
$$\sigma = \sigma_1 - j\sigma_2 = \frac{j\omega\mu_0}{Z_s^2} \quad (8)$$



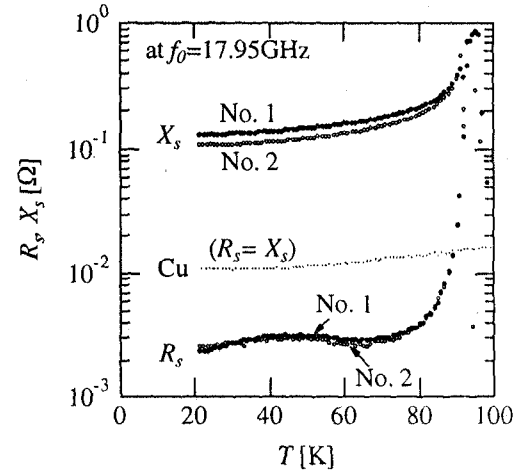
(a) $f_2 - T$ for TE_{012} mode



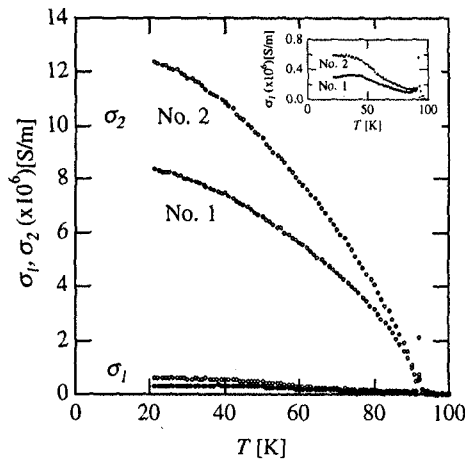
(b) $f_1 - T$ for TE_{021} mode



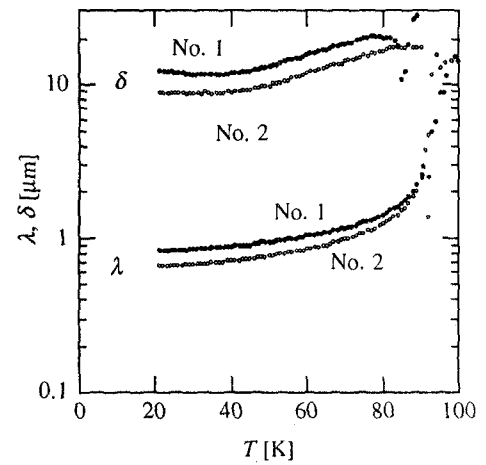
(c) $\epsilon_r - T$



(d) $R_s, X_s - T$



(e) $\sigma - T$



(f) $\lambda, \delta - T$

Fig. 3. Measured results for ϵ_r and $\tan\delta$ of a BMT ceramic rod and Z_s of melt-textured YBCO bulk plates by one-resonator method.
(Nos. 1 and 2 are for the first and second measurements, respectively.)

and the skin depth δ and the penetration length λ are given by

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma_1}} \quad (9)$$

$$\lambda = \sqrt{\frac{1}{\omega\mu_0\sigma_2}} \quad (10)$$

MEASURED RESULTS

A program was developed to measure the T dependencies of ϵ_r , $\tan\delta$ and Z_s automatically. In comparison with superconductor, at first, a BMT ceramic rod ($\epsilon_r=24$, $\tau_\alpha=6.3\text{ppm/K}$, MURATA MFG. CO.) having $D=6.994 \pm 0.001\text{mm}$ and $L=4.003 \pm 0.001\text{mm}$ [5] was placed between two copper plates of $d=35\text{mm}$, which is greater than estimated value 28mm required for measurement. The values of f_1 , Q_{u1} , f_2 and Q_{u2} were measured as a function of T . The values of ϵ_r were calculated from these measured values, where $L_0=4.0035\text{mm}$ was used in calculation, so that two ϵ_r values take the same values. Furthermore, the actual length of the rod was decided to be $L_0=4.0033\text{mm}$ by taking the skin depths of two copper plates $\delta \approx 0.2\mu\text{m}$ into account. The detail is given in [5].

Then the two copper plates were changed into two melt-textured YBCO bulk plates (IMURA MATERIAL R&D CO.) of $d=33\text{mm}$ [5]. Similarly the values of f_1 , Q_{u1} , f_2 and Q_{u2} were measured as a function of T . A similar measurement was repeated.

The f_1 and f_2 for the Cu and YBCO plates are shown in Fig. 3 (a), (b) where the Nos. 1 and 2 indicate in the curves are for the first and second measurements, respectively. The ϵ_r values calculated from these f_1 and f_2 values are shown in Fig. 3 (c). The ϵ_{act} and X_s values calculated from the ϵ_{r1} and ϵ_{r2} values are given in Fig. 3 (c) and (d), respectively. The R_s values in Fig. 3 (d) have already presented in [5]. The σ_1 and σ_2 values calculated using the measured Z_s values are shown in Fig. 3 (e).

Furthermore, λ and δ values calculated using σ_1 and σ_2 values are shown in Fig. 3 (f). The reproducibility of these measured results is considerably good.

CONCLUSION

It was verified that the one-resonator method is useful to measure the T dependence of Z_s , because they can be obtained from only one measurement for these two modes. The excellent reproducibility of measurements can be obtained, compared with the conventional two-resonator method.

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